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PION PROPERTIES IN A HOT $\pi N\Delta$ GAS

R. RAPP¹ and J. WAMBACH^{1 2}

*Department of Physics
University of Illinois at Urbana-Champaign
1110 West Green St., Urbana, IL 61801-3080, USA*

Abstract

Based on a recent meson-exchange model for the vacuum $\pi\pi$ interaction we compute selfconsistently the in-medium $\pi\pi$ scattering amplitude and pion selfenergy in a hot $\pi N\Delta$ gas. The contributions to the pion selfenergy are calculated from the $\pi\pi$ T-matrix as well as from p-wave interaction with nucleons and thermally abundant Δ 's. Results are presented for two scenarios believed to be realized in the relativistic heavy ion collisions performed at the GSI-SIS and the CERN-SpS. Possible implications for the observed soft pion enhancement at both SIS and SpS are indicated.

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¹ also: IKP (Theorie), Forschungszentrum Jülich, D-52425 Jülich, F.R.G.

² also: ITKP, Universität Bonn, D-53115 Bonn, F.R.G.

1 Introduction

Although current ultrarelativistic heavy ion collisions (URHIC's), performed e.g. at the BNL-AGS or at the CERN-SpS, presumably have not yet produced a quark-gluon plasma (QGP), they allow the study of hot hadronic matter in a temperature range of typically $T=100-200$ MeV [1]. From a theoretical point of view, a precise understanding of such highly excited matter is clearly needed to infer consequences on the equation of state beyond the nuclear matter saturation point $\rho_0 = 0.16 \text{ fm}^{-3}$ and at non-zero temperatures. From an experimental point of view, on the other hand, a quantitative description of the hadronic gas phase is essential to identify signatures of a QGP, possibly formed in the early stage of head-on collisions at the future RHIC or LHC colliders (or maybe already with the lead beam to be injected into the SpS soon).

In central collisions most of the produced particles are pions. Thus, various features of a hot interacting pion gas have been discussed extensively in the literature [2, 3, 4, 5, 6, 7, 8, 9]. However, even at the highest currently available energies of about 200 GeV/A in the laboratory frame, the midrapidity regions still show an appreciable 'baryon contamination'. According to recent studies, the $\pi\pi$ interaction in cold nuclear matter undergoes considerable modifications already well below saturation density [10, 11]; therefore one might expect a significant impact of small baryonic components on the pions even in the presence of a much higher pion density. In the present paper we shall extend our self-consistent analysis of a thermal pion gas [12] by including nucleons as well as thermally excited Δ 's. The latter seem to be of special relevance in connection with the 'soft pion puzzle'. As was shown by Brown et al. [13], pions arising from Δ decay give an important contribution to the soft component of the pion- p_T -spectra at the AGS. Furthermore, a recent analysis of SIS data [14] reports strong evidence for so-called 'resonance matter' in which about one third of the nucleons are excited into higher resonances (predominantly Δ 's). In such a scenario the baryons are strongly out of chemical equilibrium.

Our calculations are based on the assumption of thermal equilibrium which is motivated by standard estimates for the mean number of collisions per particle until freezeout. On

the other hand, also the pions may decouple from chemical equilibrium prior to freezeout. This would lead to a non-zero chemical potential μ_π which could be close to the pion mass [15]. Therefore we will also show some results for $\mu_\pi > 0$.

In sect. 2 we briefly review the formalism for the selfconsistent description of the pion gas [12] which forms the framework of the present study. In sect. 3 the model for the baryonic contribution to the pion selfenergy is discussed while sect. 4 gives results for the selfconsistent pion mean field as well as the $\pi\pi$ T-matrix in the presence of nucleons and Δ 's. We will focus on two different scenarios: (1) 'resonance matter' characterized by temperatures below 100 MeV and high baryon density $\rho_B \gtrsim 2\rho_0$ with an appreciable Δ component (SIS conditions) and (2) SpS freezeout conditions where $T \approx 150$ MeV and $\rho_B \lesssim \rho_0$. In sect. 5 we summarize our findings and indicate some consequences.

2 Selfconsistent Pion Gas

The basic ingredient for our description of the hot pion gas is the meson-exchange model for the $\pi\pi$ interaction in free space developed by Lohse et al. [16]. It is described by a T-matrix equation of Lippmann-Schwinger type, where – in contrast to ref. [16] – the Blankenbecler Sugar reduction scheme [17, 18] of the 4-dimensional Bethe Salpeter equation has been employed. After partial-wave decomposition the reduced scattering equation for given angular momentum J and isospin I reads:

$$T_{\pi\pi}^{JI}(Z, q_1, q_2) = V_{\pi\pi}^{JI}(Z, q_1, q_2) + \int_0^\infty dk \, k^2 \, 4\omega_k^2 \, V_{\pi\pi}^{JI}(Z, q_1, k) \, G_{\pi\pi}(Z, k) \, T_{\pi\pi}^{JI}(Z, k, q_2), \quad (1)$$

where $k = |\vec{k}|$ etc. ; Z is the total energy of the pion pair and $G_{\pi\pi}(Z, k)$ the two-pion propagator in the CMS frame with pions of momenta \vec{k} and $-\vec{k}$ (the Lohse et al. model also contains coupling to the $K\bar{K}$ -channel, which has been omitted for brevity in eq. (1) but which is included in the calculation). In the BbS form the vacuum two-pion propagator

is given by

$$G_{\pi\pi}^0(Z, k) = \frac{1}{\omega_k} \frac{1}{Z^2 - 4\omega_k^2 + i\eta} \quad (2)$$

with $\omega_k^2 = k^2 + m_\pi^2$. The pseudopotentials $V_{\pi\pi}^{JI}$ are constructed from an effective meson Lagrangian, where t- and s-channel ρ -exchange are the dominant contributions to the s- and p-wave interaction, respectively. This model gives a good description of the phase shifts and inelasticities up to ~ 1.5 GeV which is more than sufficient for our purposes.

The most important medium effect can be attributed to a modification of the pion propagation in the gas. At finite temperature, T , and pion chemical potential, μ_π , the in-medium single-pion propagator is given by

$$D_\pi(\omega, k; \mu_\pi, T) = [\omega^2 - m_\pi^2 - k^2 - \Sigma_\pi(\omega, k; \mu_\pi, T)]^{-1} , \quad (3)$$

where Σ_π denotes the pion selfenergy in the pion gas. Invoking the quasiparticle approximation (QPA), i.e. expanding Σ_π around the 'quasiparticle pole'

$$e_k = [\omega_k^2 + \text{Re}\Sigma_\pi(e_k, k; \mu_\pi, T)]^{1/2} \quad (4)$$

and retaining only terms up to first order,

$$\Sigma_\pi(\omega, k) \approx \Sigma_\pi(e_k, k) + \frac{\partial \Sigma_\pi(\omega, k)}{\partial \omega^2} \Big|_{e_k} (\omega^2 - e_k^2) , \quad (5)$$

enables one to calculate the in-medium two-pion propagator

$$G_{\pi\pi}(Z, k; \mu_\pi, T) = \int \frac{idk_0}{2\pi} D_\pi(k_0, \vec{k}; \mu_\pi, T) D_\pi(Z - k_0, -\vec{k}; \mu_\pi, T) \quad (6)$$

analytically. In addition, taking into account the Bose statistics of the surrounding gas, leads to the result

$$G_{\pi\pi}(Z, k; \mu_\pi, T) = \frac{1}{\bar{\omega}_k} \frac{z_k^2 (1 + 2f^\pi(e_k; \mu_\pi, T))}{Z^2 - 4\bar{\omega}_k^2} \quad (7)$$

with

$$\begin{aligned} z_k &\equiv \left(1 - \frac{\partial \Sigma_\pi(\omega, k)}{\partial \omega^2} \Big|_{e_k}\right)^{-1} \quad \text{the pole strength} , \\ \bar{\omega}_k^2 &\equiv e_k^2 + i z_k \text{Im}\Sigma_\pi(e_k, k) \quad \text{quasi pion dispersion relation} , \\ f^\pi(e_k; \mu_\pi, T) &= (\exp[(e_k - \mu_\pi)/T] - 1)^{-1} \quad \text{thermal Bose factor} . \end{aligned} \quad (8)$$

The pion selfenergy, on the other hand, is obtained from the on-shell forward scattering T-matrix as [12]

$$\Sigma_\pi(\omega, k; \mu_\pi, T) = \frac{\pi}{k} \int_0^\infty dp \frac{p}{e_p} f^\pi(e_p; \mu_\pi, T) \int_{E_{min}}^{E_{max}} dE_{cms} E_{cms}^3 T_{\pi\pi}(E_{cms}) . \quad (9)$$

Eqs. (1), (7) and (9) define a selfconsistency problem of Brueckner type which can be solved iteratively.

3 Pion Selfenergy in Hot Nuclear Matter

Nuclear matter, at finite temperature, not only consists of nucleons but – due to excitations via N-N collisions – also contains an admixture of baryonic resonances. Because of its relatively small excitation energy and high statistical degeneracy the most abundant resonance is the $\Delta(1232)$ which we will include in our gas scenario. Thus the total baryon density at given temperature T can be expressed as

$$\begin{aligned} \rho_B(\mu_N, \mu_\Delta, T) &= \rho_N(\mu_N, T) + \rho_\Delta(\mu_\Delta, T) \\ &= \int \frac{d^3q}{(2\pi)^3} [4f^N(q; \mu_N, T) + 16f^\Delta(q; \mu_\Delta, T)] \end{aligned} \quad (10)$$

with $f^a(q; \mu_a, T) = (\exp[(E_q^a - \mu_a)/T] + 1)^{-1}$ and $E_q^a = \sqrt{q^2 + M_a^2}$ for $a = N, \Delta$. Assuming chemical equilibrium ($\mu_N = \mu_\Delta$), at $T = 100$ MeV, every fourth nucleon is excited into a Δ , whereas for $T \geq 170$ MeV Δ 's are already in the majority (see Fig. 1).

To calculate the pion selfenergy $\Sigma_{N\Delta}$ in the $N\Delta$ gas, we use the finite-temperature extension of the standard particle-hole model [19]. In this approach, the most important modification of the pion propagation is attributed to p-wave excitations of the type $\alpha = ab^{-1}$ with $a, b = N, \Delta$. The amplitude for such processes is obtained from a folding integral over the particle and hole propagator resulting in the standard Lindhard functions:

$$\phi_\alpha(\omega, k) = i \int \frac{d^4p}{(2\pi)^4} [G_a^0(p_0 + \omega, \vec{p} + \vec{k}) G_{b^{-1}}^0(p_0, \vec{p}) + G_a^0(p_0 - \omega, \vec{p} - \vec{k}) G_{b^{-1}}^0(p_0, \vec{p})] \quad (11)$$

with

$$\begin{aligned}
G_a^0(p_0, \vec{p}) &= \frac{1 - f^a(p)}{p_0 - E_a(p) + i\eta} \text{ the nucleon or } \Delta \text{ propagator,} \\
G_{b-1}^0(p_0, \vec{p}) &= \frac{f^b(p)}{p_0 - E_b(p) - i\eta} \text{ the nucleon-hole or } \Delta\text{-hole propagator,} \quad (12)
\end{aligned}$$

where $E_a(p) = E_p^a - M_b - \frac{i}{2}\Gamma_a$, $E_b(p) = E_p^b - M_b + \frac{i}{2}\Gamma_b$. For the Δ width, Γ_Δ , we choose the relativistic parameterization of ref. [19] supplemented by a density dependent term to account for higher-order medium corrections. Then

$$\Gamma_\Delta(k; \rho_B) = \frac{2}{3} \frac{f_{\pi N \Delta}^2}{4\pi} \frac{q_{cms}^3}{\sqrt{s}} \frac{M_N}{m_\pi^2} \Gamma_\pi^2(k) F_{fit} + 20 \frac{\rho_B}{\rho_0} \text{MeV} \quad (13)$$

with q_{cms} being the pion momentum in the πN CMS and \sqrt{s} the πN CMS energy. The form factor $\Gamma_\pi(k) = (\Lambda_\pi^2 - m_\pi^2)/(\Lambda_\pi^2 + k^2)$ accounts for the hadronic size of the pion-baryon vertex ($\Lambda_\pi = 1200$ MeV). The density-dependent term has been estimated from pion-nucleus optical potentials. In the same spirit we take into account a width for the nucleon $\Gamma_N = 10\rho_B/\rho_0$ MeV. An additional factor $F_{fit} = 1.36$ has been introduced in order to reproduce the free space value $\Gamma_\Delta = 115$ MeV at the resonant momentum $k_{res} = 297$ MeV/c.

The second term in eq. (11) is obtained from the first one by replacing $(\omega, \vec{k}) \rightarrow (-\omega, -\vec{k})$. It characterizes the exchange graph which is necessary to ensure Bose symmetry when interchanging the in- and outgoing pion line. After a standard contour integration in the complex p_0 plane the Lindhard functions can be simplified as

$$\phi_\alpha(\omega, k) = - \int dp \frac{p^2}{(2\pi)^2} \frac{f^b(p)}{(2\pi)^2} \int_{-1}^{+1} dx \sum_{m=1}^2 \frac{1 - f_{pk}^a(x)}{\pm\omega + E_p^b - E_{pk}^a(x) + \frac{i}{2}(\Gamma_a + \Gamma_b)} \quad (14)$$

with $E_{pk}^a(x) = (M_a^2 + p^2 + k^2 + 2pkx)^{1/2}$, $f_{pk}^a(x) = (1 + \exp[(E_{pk}^a(x) - \mu_a)/T])^{-1}$. The \pm -sign corresponds to $m = 1, 2$, respectively. From the Lindhard functions we calculate the so-called pionic susceptibilities in lowest order as

$$\chi_\alpha^{(0)}(\omega, k) = \left(\frac{f_{\pi\alpha} \Gamma_\pi(k)}{m_\pi} \right)^2 SI(\alpha) \phi_\alpha(\omega, k). \quad (15)$$

The values for the spin-isospin transition factors $SI(\alpha)$ and for the pion-baryon coupling constants $f_{\pi\alpha}$ are summarized in table 1. In the latter, the following relations have

been used: $f_{\pi N\Delta} = 2f_{\pi NN}$ (Chew-Low factor [20]) and $f_{\pi\Delta\Delta} = \frac{1}{5}f_{\pi NN}$ (from the constituent quark model [21]). In evaluating the pion selfenergy it is crucial to include the short-range spin-isospin correlations between particle and hole. These correlations can also induce transitions between the various excitation modes. Therefore, using Migdal's approximation, one ends up with a system of 4 coupled equations in the fully dressed susceptibilities χ_α :

$$\chi_\alpha = \chi_\alpha^{(0)} - \sum_\beta \chi_\alpha^{(0)} g'_{\alpha\beta} \chi_\beta \quad (16)$$

with $\alpha, \beta = NN^{-1}, \Delta N^{-1}, N\Delta^{-1}, \Delta\Delta^{-1}$. For the Migdal parameters $g'_{\alpha\beta}$ we choose momentum independent values $g'_{NN} = 0.75$, $g'_{N\Delta} = g'_{\Delta\Delta} = 0.33$. The value for g'_{NN} can be deduced from Gamow-Teller resonance systematics [23]; all others are less explored, and we fix them at the classical Lorentz-Lorenz value of 0.33. Eq. (16) is then solved by matrix inversion,

$$\vec{\chi} = A_{int}^{-1} \vec{\chi}^{(0)}, \quad (17)$$

where we have defined the 4×4 interaction matrix

$$(A_{int})_{\alpha\beta} \equiv \delta_{\alpha\beta} + \chi_\alpha^{(0)} g'_{\alpha\beta}. \quad (18)$$

The total susceptibility is the sum over the 4 channels:

$$\chi_{N\Delta} = \sum_\alpha \chi_\alpha, \quad (19)$$

and the pion selfenergy is finally given by

$$\Sigma_{N\Delta}(\omega, k; \mu_N, \mu_\Delta, T) = -k^2 z_\pi^2 \chi_{N\Delta}(\omega, k; \mu_N, \mu_\Delta, T). \quad (20)$$

Here a renormalization constant z_π has been introduced to account for all processes other than the explicitly treated p-wave excitations. Migdal et al. [22] have estimated it in a dilute gas approximation to be

$$z_\pi = (1 + 4\pi\lambda\rho_B)^{-1/2}, \quad (21)$$

where $\lambda = 0.036 \text{ fm}^{-3}$.

The on-shell pion selfenergy in the $N\Delta$ gas is now obtained from a selfconsistent solution of the in-medium pion dispersion relation

$$e_k^2 = \omega_k^2 + \Sigma_{N\Delta}(e_k, k) \quad (22)$$

with $e_k^2 = \omega_k^2 + \text{Re}\Sigma_{N\Delta}(e_k, k)$, i.e. the selfconsistency is restricted to the real part of the quasi pion dispersion relation.

The results for the pion mean field (optical potential), defined as

$$U_{N\Delta}(k) \equiv \frac{\Sigma_{N\Delta}(e_k, k)}{2\omega_k}, \quad (23)$$

are displayed in Fig. 2 at various temperatures and total baryon densities. The relative abundances of nucleons and Δ 's were fixed via the condition for chemical equilibrium, i.e. $\mu_N = \mu_\Delta$. The real part of $U_{N\Delta}$ (upper panels in Fig. 2) shows a characteristic minimum induced by resonant ΔN^{-1} excitations. The latter also show up in the imaginary part exhibiting a rather pronounced peak at the resonant pion momentum of about 300 MeV/c (lower panels). At higher temperatures the increased thermal motion of the nucleons leads to a slight downward shift of the peak and we also observe an overall reduction in the absorption, which implies a decrease in $\text{Re}U_{N\Delta}$ as well. This effect is directly related to the relative decrease of the nucleon component in the gas (see also Fig. 1). Thus the resonant ΔN^{-1} excitations are suppressed, partly because there are relatively fewer nucleons and partly due to stronger Pauli blocking of possibly excited Δ 's. On the other hand, the pion interaction with the thermally abundant Δ 's is essentially non-resonant. We would also like to mention that the selfconsistent solution of eq. (22) is not always unique. Starting from nucleon densities $\rho_N \approx 1.5\rho_0$ (slightly depending on temperature and Δ density) the ΔN^{-1} branch, and eventually also the NN^{-1} branch, begin to mix so strongly that additional solutions appear. Physically this can be understood as an explicit splitting of the pion propagation into different modes.

4 The $\pi N\Delta$ Gas

We now extend the formalism described in sect. 2 for a pure pion gas by including the baryonic selfenergy contribution $\Sigma_{N\Delta}$. Then the total pion selfenergy receives two contributions:

$$\Sigma_{\pi N\Delta}(\omega, k; \mu_\pi, \mu_N, \mu_\Delta, T) = \Sigma_\pi(\omega, k; \mu_\pi, T) + \Sigma_{N\Delta}(\omega, k; \mu_N, \mu_\Delta, T) . \quad (24)$$

Consequently, in eqs. (3)-(8), Σ_π has to be replaced by $\Sigma_{\pi N\Delta}$, such that

$$G_{\pi\pi}(Z, k; \mu_\pi, \mu_N, \mu_\Delta, T) = \frac{1}{\bar{\omega}_k} \frac{z_k^2 (1 + 2f^\pi(e_k; \mu_\pi, T))}{Z^2 - 4\bar{\omega}_k^2} \quad (25)$$

with

$$\begin{aligned} z_k &= (1 - \frac{\partial \Sigma_{\pi N\Delta}}{\partial \omega^2}|_{e_k})^{-1} , \\ \bar{\omega}_k^2 &= e_k^2 + iz_k \text{Im} \Sigma_{\pi N\Delta}(e_k, k) , \\ e_k^2 &= \omega_k^2 + \text{Re} \Sigma_{\pi N\Delta}(e_k, k) . \end{aligned} \quad (26)$$

Therefore eqs. (1), (9), (20) and (25) form our extended set of selfconsistent equations, which we solve by numerical iteration. The pion density $n_\pi(\omega_k; \mu_\pi^{(0)}, T)$ is kept fixed by readjusting μ_π after each iteration step according to the change in the quasi pion energy e_k such that, after convergence, $n_\pi(\omega_k; \mu_\pi^{(0)}, T) = n_\pi(e_k; \mu_\pi^*, T)$ with the appropriate value for μ_π^* .

Let us first concentrate on a scenario which may be realized at SIS energies. A recent compilation of data [14] gives strong evidence for 'resonance matter', i.e. hot nuclear matter with an abundance of nucleonic resonances in excess of values obtained for chemical equilibrium; at SIS temperatures of typically $T = 75$ MeV the relative abundance of Δ 's in chemically equilibrated nuclear matter ($\mu_N = \mu_\Delta$) would be about 10% (see also Fig. 1); however, the analysis of various data in conjunction with BUU calculations suggests a nucleon-to-delta ratio of about 2:1 [14]. Fig. 3 shows our selfconsistent results for the on-shell $\pi\pi$ T-matrix $T_{\pi\pi}(Z)$ (upper panels) as well as the pion optical potential $U_{\pi N\Delta}(k)$ (middle panels) at $T = 75$ MeV, $\rho_B = 2\rho_0$ and for various nucleon-to-delta ratios (fixed

by independent choice of μ_N and μ_Δ). For the pion chemical potential we choose $\mu_\pi^{(0)} = 0$. At $T = 75$ MeV this corresponds to a pion density of 0.01 fm^{-3} which is too small to show any visible effect on $T_{\pi\pi}$ or $U_{\pi N\Delta}$. The optical potential is dominated by the ΔN^{-1} excitation channel. Due to the selfconsistency requirement the lowering of the quasi pion energy leads to an upward shift of the maximum in $ImU_{\pi N\Delta}$ which now peaks above the resonant momentum of $k = 297 \text{ MeV}/c$ in free space. As in the previous section, we see that an increase of the Δ abundance at fixed total baryon density reduces the magnitude of the pion mean field. The in-medium $\pi\pi$ T-matrix in the σ -channel (upper-left part in Fig. 3) as well as the ρ -channel (upper-right part) shows a strong reduction of the peak values as compared to the vacuum. The depletion of strength in the σ -channel in the energy range $400 \text{ MeV} \leq Z \leq 800 \text{ MeV}$ comes from the rapid variation in $ReU_{\pi N\Delta}$ for pion momenta $k \approx 250 - 400 \text{ MeV}/c$, turning from strong attraction to repulsion, thus suppressing pion modes of energies $e_k \approx 200 - 400 \text{ MeV}$. In the same momentum region, the real part of $\partial\Sigma_{\pi N\Delta}/\partial\omega^2|_{e_k}$ acquires large positive values (lower-left panel in Fig. 3) which numerically further enhances the depletion effect. This mechanism is also responsible for strength accumulation slightly below the $K\bar{K}$ threshold at $Z \approx 970 \text{ MeV}$. We furthermore observe considerable accumulation of strength below the two-pion threshold. To decide on whether two-pion bound states are formed requires an analysis beyond the quasiparticle approximation (QPA) taking into account the off-shell behavior of the pion selfenergy in more detail [24]. Besides broadening, the ρ -resonance (upper-right panel) shows a considerable upward shift of about $70-90 \text{ MeV}$. The same qualitative features have also been found for cold nuclear matter in ref. [11] where a similar ansatz for the pion selfenergy has been used neglecting Fermi motion, however. Furthermore, the 2-pion propagator was calculated in a three-branch model, whereas in the present paper the quasiparticle picture is employed. The validity of the QPA is limited to rather small pion widths, $\Gamma_k = -2ImU_{\pi N\Delta}(k)$. It may therefore be questioned for nucleon densities above $\sim 1.5\rho_0$ since the ratio Γ_k/e_k is already close to one in the peak region, $k \approx 300 \text{ MeV}/c$. However, we have checked the sensitivity of our results with respect to different

orders in the expansion of the pion selfenergy by performing calculations with purely real pole strength, i.e neglecting the imaginary part of $\partial\Sigma_{\pi N\Delta}/\partial\omega^2|_{e_k}$. It turns out that these results coincide with those for complex z_k within a few percent – in contrast to the pure pion gas analysis of ref. [12] where a strong dependence on $Im\partial\Sigma_{\pi}/\partial\omega^2|_{e_k}$ was found. Thus one might be encouraged to conclude that the QPA produces meaningful results even at baryon densities as high as $\rho_B \approx 2\rho_0$.

The second scenario which we would like to discuss is believed to be realized in the midrapidity regions of the SpS experiments at CERN. In central collisions at an energy of about 200 GeV/A in the lab frame, the two colliding nuclei almost pass through each other while depositing large amounts of energy in the central zone. After hadronization one should then encounter a hot and dense gas of mesons contaminated with a net baryon density of about $0.5\rho_0$ as extracted from measured multiplicities [25]. The predominant meson species is the pion. At a temperature of $T = 150$ MeV and in chemical equilibrium ($\mu_{\pi} = \mu_K = \mu_{\bar{K}} = 0$) the kaon-/ antikaon density is already less than $\frac{1}{6}$ of the pion density. For a hot $\pi N\Delta$ gas at $T = 150$ MeV we again perform selfconsistent calculations as described in the beginning of this section. This time, the gas is assumed to be in chemical equilibrium characterized by the condition

$$\mu_{\pi}^{(0)} + \mu_N = \mu_{\Delta} . \quad (27)$$

Thus we end up with three independent thermodynamic parameters: the temperature T , the pion density $n_{\pi}(\mu_{\pi}^{(0)}, T)$ – which, at a given temperature, is fixed by a starting value of the pion chemical potential – and the total baryon density ρ_B . At given T and $\mu_{\pi}^{(0)}$ the relative abundances of nucleons and Δ 's are then determined from eq. (27). The results for the in-medium $\pi\pi$ T-matrix and the pion optical potentials are displayed in Fig. 4 for different values of $\mu_{\pi}^{(0)}$ and ρ_B . Even at rather low baryon densities of $\rho_B = 0.5\rho_0$ and at $\mu_{\pi}^{(0)} = 0$ one clearly recognizes the signatures of the baryonic selfenergy contribution (which are even more pronounced for $\rho_B = 1.0\rho_0$): the Δ -peak in $ImU_{\pi N\Delta}$ at $k \approx 300$ MeV/c (lower-middle part in Fig. 4) and a corresponding minimum in $ReU_{\pi N\Delta}$ at $k \approx 200$ MeV/c (lower-left part) as well as an appreciable reduction of the T-matrix

peak values in both the σ - and ρ -channel (upper-left and -middle part). However, the attraction in $ReU_{\pi N\Delta}$ is much less than under SIS conditions. As a consequence two-pion quasi bound states in the s-wave now appear close to the two-pion threshold. The impact of the pionic gas component becomes apparent, if we increase $\mu_\pi^{(0)}$ from zero to 100 MeV which is equivalent to an increase in pion density from 0.12 fm^{-3} to 0.27 fm^{-3} . The optical potentials now exhibit considerable non-zero values at zero momentum. In $ImU_{\pi N\Delta}$ this is essentially caused by the s-wave $\pi\pi$ interaction which is enhanced due to large accumulation of strength in the threshold region of $ImT_{\pi\pi}^{00}$ (upper-left part of Fig. 4). This threshold enhancement is a consequence of the Bosefactors $(1 + 2f^\pi)$ in $G_{\pi\pi}$ leading to a stronger weighting of small momenta. The downward shift of the in-medium two-pion threshold is due to the decrease in the pion mass of nearly 20 MeV, generated by $ReU_{\pi N\Delta}(k=0)$. At $\mu_\pi^{(0)} = 100 \text{ MeV}$, the resonances in the different JI -channels show the remarkable feature that they seem to accumulate the major part of the strength in the corresponding partial wave. In the s-wave this is the case for the 2π bound state, the resonant structure near the $K\bar{K}$ threshold as well as the $\epsilon(1400)$ (which is now shifted to $Z \approx 1150 \text{ MeV}$). In the p-wave the ρ peak becomes strongly enhanced (upper-middle part of Fig. 4) and the same is observed for the d-wave $f_2(1270)$ in $ImT_{\pi\pi}^{20}$ (upper-right part of Fig. 4). In ref. [12] it was shown that these enhancements are related to the energy dependence of the absorption (lower-right part of Fig. 4).

From the partial-wave decomposed T-matrices we can compute the in-medium total cross sections for $\pi^+\pi^-$ scattering as well as for the charge-exchange reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$. This is done by a straightforward transformation of the T-matrix from the isospin to the particle basis [16] :

$$\begin{aligned}
\langle \pi^+\pi^- | T_{\pi\pi}^J | \pi^+\pi^- \rangle &= \frac{2}{3} T_{\pi\pi}^{J0} + T_{\pi\pi}^{J1} + \frac{1}{3} T_{\pi\pi}^{J2} , \\
\langle \pi^0\pi^0 | T_{\pi\pi}^J | \pi^+\pi^- \rangle &= \frac{\sqrt{2}}{3} T_{\pi\pi}^{J0} - \frac{\sqrt{2}}{3} T_{\pi\pi}^{J2} .
\end{aligned} \tag{28}$$

The total cross section is then obtained as

$$\sigma_{tot}^{\pi^+\pi^-\rightarrow\pi\pi}(Z) = \frac{\pi^3}{4} Z^2 \sum_J (2J+1) |\langle\pi\pi|T_{\pi\pi}^J(Z)|\pi^+\pi^-\rangle|^2, \quad (29)$$

where the final state consists of either two charged or two neutral pions. Fig. 5 summarizes our cross section results for the CERN conditions. For both charged and neutral pions in the final state the in-medium cross section is enhanced over the vacuum results for $Z \geq 800$ MeV as soon as $\mu_\pi^{(0)}$ reaches about $m_\pi/2$ (at $T = 150$ MeV, $\rho_B = 0.5\rho_0$). Especially the $f_2(1270)$ -resonance seems to be quite sensitive to this enhancement mechanism, which might show up in future dilepton or two-photon measurements in form of a resonance narrowing. On the other hand, there is a suppression of $\sigma_{tot}^{in-medium}$ in the intermediate energy range $300 \text{ MeV} \leq Z \leq 700 \text{ MeV}$ where, in our model, there is no genuine $\pi\pi$ resonance. For chemically equilibrated pions at $T = 150$ MeV this suppression can be quite large, especially at higher ρ_B . A similar suppression is found, in this case, for the ρ peak region (upper part of Fig. 5).

5 Summary

Starting from a realistic model for the $\pi\pi$ interaction in free space we have examined medium modifications of the $\pi\pi$ T-matrix and the single-pion dispersion relation in a hot gas of pions, nucleons and Δ 's. In evaluating the in-medium $\pi\pi$ T-matrix we have taken into account statistical (Bose factors) as well as dynamical (pion selfenergy) effects on the 2π propagator in the scattering equation. To simplify the numerical treatment we have applied the QPA. The pion gas contribution to the selfenergy was calculated from the $\pi\pi$ T-matrix while, for the baryonic component, a standard treatment using finite-temperature Lindhard functions has been adopted. We have solved the resulting selfconsistency problem by numerical iteration. In view of possible applications we have focused on two different scenarios corresponding to the URHIC's performed at the GSI-SIS and at the CERN-SpS.

For the SIS scenario, characterized by baryon densities $\rho_B \approx 2 - 3\rho_0$ with about $1/3 - 1/2$ of the nucleons excited into Δ 's and by very low pion density, we find a strong reduction in the imaginary part of the in-medium $\pi\pi$ T-matrix. The pion optical potential exhibits considerable attraction for pion momenta $k \approx 200 - 300$ MeV/c which is generated by resonant ΔN^{-1} excitations. Increasing the relative Δ abundance is shown to reduce this attraction somewhat because of a corresponding decrease of the nucleon abundance at fixed ρ_B . As was recently pointed out by Koch and Bertsch [26], such an attraction is a promising candidate to explain (at least partially) the low- p_T enhancement in the pion p_T -spectra observed at SIS: assuming an adiabatically expanding hadron gas, the heavier nucleons move at a much smaller velocity than the pions; thus even low-momentum pions may 'escape' from the nucleonic component of the fireball, in which case the above mentioned attraction should lead to a significant softening.

The SpS scenario, on the other hand, is characterized by much higher temperatures of $T \approx 150$ MeV and relatively small baryon densities $\rho_B \lesssim \rho_0$. As a consequence, the attraction in the pion mean field is much smaller than in the high baryon density case, even though some of it is restored by the increase in pion density. This additional attraction, however, is not able to account for the observed pion low- p_T enhancement in the SpS experiments [26]. As long as we assume the pions to be in chemical equilibrium, the imaginary part of the $\pi\pi$ T-matrix is reduced overall as compared to the vacuum case, although less pronounced than under SIS conditions. For finite pion chemical potentials, $\mu_\pi \gtrsim m_\pi/2$, the various partial-wave channels of the T-matrix become enhanced over the free space results, however, especially in vicinity of the resonances. As suggested by numerical simulations of the bosonic Boltzmann equation [8], such an enhancement could probably lead to thermalization of the pionic gas component and hence generate an excess of low- p_T pions through the $(1 + f^\pi)$ factors in the collision integral. One should be cautious, however, in trusting the QPA beyond $T = 150$ MeV and for high pion chemical potentials, since the off-shell properties of Σ_π (especially in the imaginary part) become important.

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TABLES

α	NN^{-1}	ΔN^{-1}	$N\Delta^{-1}$	$\Delta\Delta^{-1}$
$SI(\alpha)$	4	16/9	4/9	400
$f_{\pi\alpha}^2/4\pi$	0.081	0.324	0.324	0.00324

Table 1: *Spin-isospin transition factors and coupling constants for pion induced p-wave particle-hole excitations in a hot $N\Delta$ gas.*

FIGURE CAPTIONS

Fig. 1: Relative abundances of nucleons (full line) and Δ 's (dashed line) in chemical equilibrium ($\mu_N = \mu_\Delta$) at a total baryon density of $\rho_B = \rho_0$.

Fig. 2: Selfconsistent pion mean fields in a hot $N\Delta$ gas in chemical equilibrium ($\mu_N = \mu_\Delta$) at two different total baryon densities:
 upper part: real part of the pion mean field;
 lower part: imaginary part of the pion mean field;
 (long-dashed lines: $T = 100$ MeV, short-dashed lines: $T = 150$ MeV, dotted lines: $T = 200$ MeV).

Fig. 3: Selfconsistent $\pi N\Delta$ gas at SIS conditions ($T = 75$ MeV, $\rho_B = 2\rho_0$, $\mu_\pi^{(0)} = 0$):
 upper part: on-shell $\pi\pi$ T-matrix in σ -channel (JI=00) and ρ -channel (JI=11);
 middle part: real and imaginary part of the pion mean field;
 lower part: real and imaginary part of the energy derivative of $\Sigma_{\pi N\Delta}$;
 (long-dashed lines: nucleon-to-delta ratio N: Δ =4:1, short-dashed lines: N: Δ =2:1, dotted lines: N: Δ =1:1; the full lines in the upper part correspond to the $\pi\pi$ T-matrix in free space).

Fig. 4: Selfconsistent $\pi N\Delta$ gas at CERN conditions ($T = 150$ MeV, $\mu_N + \mu_\pi^{(0)} = \mu_\Delta$):
 upper part: on-shell $\pi\pi$ T-matrix in σ -channel (JI=00), ρ -channel (JI=11) and f_2 -channel (JI=20);
 lower part: pion mean field and imaginary part of the energy derivative of $\Sigma_{\pi N\Delta}$;
 (long-dashed lines: $\mu_\pi^{(0)} = 0$ and $\rho_B = 0.5\rho_0$, short-dashed lines: $\mu_\pi^{(0)} = 0$ and $\rho_B = \rho_0$, dotted lines: $\mu_\pi^{(0)} = 100$ MeV and $\rho_B = 0.5\rho_0$; the full lines in the upper part correspond to the $\pi\pi$ T-matrix in free space).

Fig. 5: $\pi\pi$ cross sections in a $\pi N\Delta$ gas at CERN conditions ($T = 150$ MeV, $\mu_N + \mu_\pi^{(0)} = \mu_\Delta$):
upper part: $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering;
lower part: $\pi^+\pi^- \rightarrow \pi^0\pi^0$ charge-exchange reaction;
(line identification as in upper part of Fig. 4 except that the dotted lines are now
for $\mu_\pi^{(0)} = 75$ MeV).

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